Pricing of an Endogenous Peak-Load

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Abstract

This paper aims to explore the peak-load price results arising in a congestion game setting. A continuum of players decide when consuming a service (say, during the day or the night): for instance, they choose when connecting to internet or when driving a car in the road network. The utility they get is a function of the individual preferences and of the aggregate behavior of the other players. Therefore day and night demands are endogenous. We consider what prices a social planner imposes to drive the players' choice towards the equilibrium distribution of first best. Moreover, we consider what prices (and what distribution) a monopolist, maximizing his profit, chooses, when he has to satisfy an universal service requirement and when he may restrict his supply. Finally, we determine which capacity level is optimal to install for either a social planner or a monopolist in this setting.

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1 Introduction

In the literature about peak-load pricing, the demand functions for both the peak and the off-peak periods are, very often, exogenously given and independent (Boiteux, 1951, and Steiner, 1957, were the first to study the peak-load pricing and assumed these features for the demand functions). Therefore, the peak period (i.e. the period with the highest demand) is exogenously determined.

The well known result of these studies is an efficient pricing rule which prescribes that individuals choosing the peak period pay for both operating and fixed costs and those choosing the off-peak pay only for the variable costs (see Crew, Fernando, and Kleindorfer, 1995, for a recent survey).

Nevertheless, this framework is static while, actually, at least one fraction of the consumers is free to decide when consuming. In fact, Shy, 2001, considers a dynamic OLG model where the agents decide weather consuming today, tomorrow or never. In such a way, he endogenously determines the demand functions and what the peak period is.

In this paper, we develop the same intuition of Shy, but we wish to focus mainly on the weak strategic interaction between consumers that, we believe, plays a crucial role in the consumption choices.

For instance, it is an usual experience to postpone the internet connection when we observe long delays before getting the desired results. Often, such delays are due to the high number of users connected and to the high number of requests. Thus, we suffer either a simple disutility or the constraint to postpone our action, because of the others' consumption decisions.

There is also an indirect transmission system for these externalities, via prices, because of the nature of the cost function in the markets where a problem of peak-load arises. In the literature, it is always well highlighted that the peak load problem comes from the impossibility of storing the good or the service during the low demand period to make it available during the high demand (Crew, Fernando, and Kleindorfer, 1995, and Baumol and Faulhaber, 1988, among others).

This storing impossibility leads to a production process that should be able to satisfy the demand in real time. Such a constraint implies, roughly speaking, a step and increasing cost function. To satisfy a high demand it is necessary to operate at high production costs and therefore the prices have to be higher in order to yield a nonnegative profit.

Indeed, the decision of a consumer has an externality on the others: when he consumes, he may increase the prices for all the individuals, deciding to consume at the same period. Alternatively and more interestingly, if a consumer changes his choice, he will induce the price of his newly chosen period to increase and, simultaneously, he will induce the price of his previously chosen period to decrease.

Knowing the existence of such externalities, a rational agent may want to act strategically in order to maximize his own utility, i.e. he may want to take into account what the others do, before choosing when consuming. This feature makes the choice of any individual (i.e. the demand for the service in either periods) endogenous as well as when the peak arises. Actually, demand results from the simultaneous interaction of the agents. The highest demand determines the peak period.

Let us simply call congestion the global influence of the others. Congestion makes our framework a game where it is possible to determine, with or without prices, a players' distribution of (Nash) equilibrium. This is because it represents the aggregate behavior of the opponents, or a function of their choices.

Here are few examples where congestion plays a role. In the internet connection, the time necessary to reach the desired web page and to receive an answer increases as the number of users connected at the same time increases. In the phone service, where the lines are dedicated to pair-communications, it may be necessary to wait a while before taking the line during the peak hours. In the road network, the congestion effect is the traffic and the time we spend in car (and so our satisfaction) depends on the traffic level.

As mentioned, another effect is more indirect. It is associated with the price growth due to an high demand in a service presenting an increasing cost function. In the deeply studied electrical market (see Borenstein and Bushnell, 2000, for a recent contribution), during the peak periods some additional gas power plants work to quickly satisfy the high demand. Such plants have relatively low installation but high marginal costs. Thus, to cover the costs, the electricity producers have to increase the peak period price. Indeed, the cause of the high price is the concentration of the demand at the same time.

Other similar situations can be found in the airline transport, in the hotel service, in the airport services and in the mail system, where considerations of peak-load pricing are jointly solved with price and quality discrimination objectives (Crew, Fernando, and Kleindorfer, 1995). Notice that, often, quality is for how fast the service is: examples are the priority land services in the airports for the business class travelers and the first class or priority mail compared with the second class or ordinary.

In terms of game theory, the player t's payoff decreases with the number of other players that chooses the same period of t, i.e. this is a game with rivalry (see Konishi, Le Breton, and Weber, 1997).

Let us to keep the internet example, to discuss our issues in a more apparent way.

Each agent compares the payoff he gets by connecting now or later. Suppose that a social planner or a monopolist introduces prices, different for the two times. Such prices not only allow to efficiently allocate the costs among the players, but they may also modify the relative payoff of each agent, directing their choice.

Hence, prices become a tool to distribute the demand in the time efficiently (i.e. separating the agents on the basis of their willingness to pay for consuming in either period, taking into account, at the same time, the congestion effect).

In this paper we mainly focus on the relation between prices and players' distribution, on what prices a social planner sets to induce the first best allocation and on what prices a monopolist sets in order to maximize his profits. We prove that there is a one-to-one relation between prices and players' distribution in each case, but the resulting allocations may be very different.

This framework is, from a technical point of view, very different from that of Shy, 2001, and, conceptually, the main difference is the concern on prices. These can completely determine any equilibrium distribution and they embody the congestion effects. Such effects are explicit rather than implicit as in Shy.

We assume that there are no variable costs. This is the case for instance of internet where operating and fixed costs are almost negligible (see McKnight and Bailey, 1997) or the case of a urban road network, where the maintenance and building costs are payed with public funds. Moreover, except for section 5, we assume also no fixed costs.

Therefore, differently from the traditional studies, the prices role is purely distributive and does not deal with the cost covering. Thus, we can not directly compare the prescription of the peak-load pricing literature about the allocation of the fixed cost between the peak and the off-peak consumers.

However, if we imagine that some fixed costs exist but are low enough (such that the budget balance constraint is slack), we obtain that they are completely payed by the peak period consumers, as in the traditional literature (Steiner, 1957, among others).

On the contrary, in section 6, we add installation (fixed) costs. We are interested in determining the optimal capacity supply jointly with either a social welfare or a private profit maximization. In this case a punctual comparison with the habitual pricing rule is possible.

We believe that our approach is interesting for at least two reasons. First, it recognizes that, usually, peak and off-peak demands are endogenous, i.e. there may be some individuals that switch between periods, depending on the prices level, without renounce to their consumption. In the traditional literature the demand functions are assumed exogenous: thus, increasing a price, or both, simply reduces demand. Here, roughly speaking, demand moves and does not disappear.

Second, our approach leads us to completely determine the relation between prices and agents' distribution, allowing us to compare what different objective functions (social surplus or private profit) imply about individuals' distribution and the aggregate loss of welfare due to congestion.

Let us now present how the paper is organized. In section 2, we describe the game and prove the existence and the uniqueness of the Nash equilibrium without prices. We also observe that, given the model formulation, the equilibrium strategy profile is completely characterized by a pivotal individual that shares the demand in two parts (day and night demand).

In section 3, we analyze how a social planner would allocate the demand in order to maximize the social surplus. We get a result in terms of the pivotal individual. He is such that his utility loss (due to the price introduction and, so, to the new relative convenience of any available strategy) equals the aggregate externality he imposes to the others, moving from his spontaneous to his induced-by-prices choice.

In section 4, we look to a profit maximizer monopolist, who sets the prices in two different contests: in the first, he has to guarantee a nonnegative utility to any consumer; in the second, he may reduce the consumers' access. We assume that day and night valuations are negatively correlated in our society. Again we obtain results in terms of the pivotal individual.

Without access reduction, the monopolist sets the prices in such a way that the pivotal individual presents the equality between his utility loss and the aggregate surplus variation the monopolist may extract from the other agents. This difference is created by the passage from the pivotal individual's spontaneous choice to his induced-by-prices choice. It results both from a variation of the marginal willingness to pay in the set of the players choosing either period, and from a change of the congestion.

With access reduction, the monopolist sets the prices in order to determine two pivotal individuals. Such individuals bound and characterize the sets of those players choosing day and night. They are such that if the monopolist renounces to serve them (and so he renounces to the price they pay), he gains the same amounts by extracting the surplus variation of both groups, generated by the reduced access.

Section 5 deals with the problem of an optimal capacity supply. Here we suppose day and night valuation positively correlated among the population: there exist people having a high valuation for day and night connection, or a low valuation for both. Again, we consider the problem of a benevolent social planner and that of a profit maximizing monopolist.

In the first setting, we determine two pivots such that marginal social benefits and costs are exactly offset, where the social costs embodies the capacity costs. This condition implies that the first best distribution is more equilibrate than the “spontaneous” distribution obtained without planner’s involvement.

In both settings we get that fixed costs are payed by the peak consumers, as in the traditional peak-load literature. What is interesting is that, in monopoly, the price peak-reverse phenomenon may arise (Bailey and White, 1974, and Shy, 2001), i.e., off-peak exceeds the peak price.

Finally section 6 concludes.

2 The game

Consider a nonatomic, anonymous, static game with complete information, as defined by Rath, 1992. There is a set of players represented by the unit interval $T = [0,1]$ in $\Re$. We assume that this interval is endowed with the Borel σ-algebra $B([0,1])$ and with the Lebesgue measure λ. Therefore, $(T, B([0,1]), \lambda)$ is both a measure and a probability space, since $\lambda([0,1]) = 1$.

The action space is simply the binary set $A = \{(1,0),(0,1)\}$ composed by the two unit vectors in $\Re^2$. In words, $(1,0)$ is for the connection at period 1 and $(0,1)$ is for the connection at period 2. In the sequel, it will be clear why we use, like Rath, this formulation. Moreover, we will often employ *strategy 1* (or day) and *strategy 2* (or night) to refer respectively to the unit vector $(1,0)$ and $(0,1)$.

Let $f : T \rightarrow A$ be the pure strategy profile and F the set of all the possible pure profiles f, i.e. $f \in F$ assigns to each player $t \in T$ a pure strategy $(a, 1-a) \in A$ with $a = 0, 1$.

We call S the set of the Lebesgue integrals of all functions $f \in F$. Notice that, by definition, the Lebesgue integral of any f represents the distribution of the players over the two pure strategies (day and night) and it can be write as a vector (q_1, q_2) with $q_i \in [0, 1]$ for $i = 1, 2$ and $q_1 + q_2 = 1$. Therefore, the set S is nothing else that the unit simplex in $\Re^2$.

Each player is endowed with a private valuation for both periods, i.e. each player is endowed with the vector $(V_1(t), V_2(t))$. Moreover, we suppose that $V_i : T \rightarrow \Re$ are positive and continuous functions, for $i = 1, 2$. To avoid trivial results, we assume that this two functions have a (unique) intersection point internal in T, and that $\Delta V(t) = V_1(t) - V_2(t)$ is a strictly increasing function.

In this setting, the anonymity of the game means that the players' payoff functions depend on the Lebesgue integral of the strategy profile. Two consequences have to be pointed out: first, each player's action has a negligible effect on the others, because a single individual forms a zero-measure set; second, what matters for a player, in order to make his choice, is simply the distribution of the others and not what any single agent does.

Indeed, the payoff function $u : T \times A \times S \rightarrow \Re$ are specified as $u_i(t, q_i) = V_i(t) - h(q_i)$ for $i = 1, 2$, representing, as usual, the two available pure strategies and q_i representing the *i-th* coordinate of $\int_T f d\lambda$. We assume that $h : [0, 1] \rightarrow \Re^+$ is continuous and increasing. Therefore the u is continuous in $T \times A \times S$. The function h represents the effect of the strategic interaction on the players' payoff and on the their choice at equilibrium.

Given the specification of u_i and the fact that h is increasing, this is a game of congestion or of rivalry, i.e. the t payoff is higher the lower is the measure of the set of the players choosing the same period of agent t. Indeed, $-h(q_i)$ is a measure for the congestion.

Assumption 1 At least one of the actions available for the players yields a positive payoff, for any (q_1, q_2).

This assumption is absolutely weak and its meaning is simply that the service has an economic value at each distribution.

We are interested in determining the Nash equilibrium of this game.

Definition 1 *A pure strategy Nash equilibrium of this game is a pure strategy profile f^* such that for almost all $t \in T$, $u(t, f^*(t), \int_T f^* d\lambda) \geqslant u(t, a, \int_T f^* d\lambda)$ for all $a \in A$.*

Proposition 1 *In this game there exists a unique Nash Equilibrium in pure strategies and it can be written as*

$$f^*(t) = \begin{cases} 2 & \textit{for } t < t^{nash} \\ 1 & \textit{for } t \geqslant t^{nash} \end{cases}$$

Proof. Following Rath, 1992, we first determine the best reply correspondence, then we consider an auxiliary correspondence obtained as the Lebesgue integral of the best reply and we look for a fixed point. If it exists, then it is possible to find a pure strategy profile which satisfies the definition 1. In this simplified setting the Lebesgue integral of the best replay correspondence is a binary vector with the *i-th* coordinate ($i = 1, 2$) equal to the Lebesgue measure of the players' set preferring the *i-th* strategy. Hence, the best reply function is

$$B_t(q_1, q_2) = \begin{cases} 1 & \text{if } u_1(t, q_1) \geqslant u_2(t, q_2) \\ 2 & \text{otherwise} \end{cases}$$

Given the formalization of the strategies described above, the best reply function (of t) can also be represented as the pair of characteristic functions $\Gamma = (1_{T1}(t), 1_{T \backslash T1}(t))$, where the set $T1$ is the set $\{t \in T, \quad u_1(t, q_1) \geqslant u_2(t, q_2)\}$, i.e. the set of all those players that prefer the strategy 1, given the strategy profile f such that $\int_T f d\lambda = (q_1, q_2)$. It is clear that the Lebesgue integral of $(1_{T1}(t), 1_{T \backslash T1}(t))$ is $(\lambda(T1), \lambda(T \backslash T1))$. It is, thus, simple to determine the condition for a fixed point of Γ, i.e.:

$$\lambda(T1) = q_1$$

$T1$ is equivalent to $\{t \in T, \quad \Delta V(t) \geqslant h(q_1) - h(q_2)\}$. Since $\Delta V(t)$ is an increasing function and it is continuous, the set $T1$ is the interval $[t^{nash}, 1]$. We can think to $\Delta V(t)$ as to a stochastic variable. Its probability law is nothing else that the measure induced by the function $\Delta V(t)$, i.e. $\lambda_{\Delta V}(A') = \lambda(\Delta V^{-1}(A'))$ is the probability of $\Delta V(t) \in A'$.

Now, let us define the cumulative distribution of ΔV as

$$F(\nu) = \lambda_{\Delta V}([-\infty, \nu]) = \lambda(\{t \in T, \ \Delta V(t) \leqslant \nu\})$$

This function is continuous and strictly increasing because $\Delta V(t)$ is. We can rewrite the fixed point condition as

$$F(h(q_1) - h(q_2)) = 1 - q_1$$

Now, $F(h(q_1)-h(1-q_1))$ is valued over $[0,1]$ and it is continuous and increasing in q_1 because $h(\cdot)$ and $F(\cdot)$ are both continuous and strictly increasing. Since $1 - q_1$ is also valued on $[0,1]$ and it is decreasing, there exists a unique pair (q_1^*, q_2^*) satisfying the equality. Therefore, this is the unique fixed point. To conclude, since $\Delta V(t)$ is increasing, the strategy profile of equilibrium f^*, i.e. the strategy profile such that $\int_T f^* d\lambda = (q_1^*, q_2^*)$, is unique and can be written as

$$f^*(t) = \begin{cases} 2 & \text{for } t < t^{nash} \\ 1 & \text{for } t \geqslant t^{nash} \end{cases}$$

where t^{nash} is such that $\Delta V(t^{nash}) = h(q_1^*) - h(q_2^*)$. ■

Two remarks are needed. First, the agent labeled t^{nash} is a pivotal individual that shares the strategy profile and fully determines the players' distribution on the two periods. Second, the economic interpretation of the implicit condition for t^{nash} is included in the following:

Proposition 2 *The pivotal individual t^{nash} is such that his payoff is exactly the same for both possible periods, or t^{nash} is the indifferent agent, given the strategy profile of equilibrium f^*.*

Proof. Simply rearranging $\Delta V(t^{nash}) = h(q_1^*) - h(q_2^*)$, we obtain

$$u_1(t^{nash}, q_1^*) = u_2(t^{nash}, q_2^*) \tag{1}$$

■

Finally, pay attention to a feature that will be very useful in the sequel. Given the form of $T1$ and the fixed point condition $\lambda(T1) = q_1$, in equilibrium we have that $1 - t^{nash} = q_1^*$ or simply

$$t^{nash} = q_2^*$$

3 The planner's solution

Consider now a simple two stage game, where a social planner aims to maximize the utilitarian social welfare, choosing in the first stage two prices P_1 and P_2 for respectively period 1 and period 2. In the second stage the continuum of players $[0, 1]$ decides in what period connecting to the service, observing the planner's prices.

Firstly, we discuss the link between the prices and the Nash Equilibrium distribution. We simply apply the proof of proposition 2. The best reply function becomes:

$$B_t(q_1, q_2) = \begin{cases} 1 & \text{if } u_1(t, q_1) - P_1 \geqslant u_2(t, q_2) - P_2 \\ 2 & \text{otherwise} \end{cases}$$

The set $T1$ is $\{t \in T, \quad \Delta V(t) \geqslant h(q_1) - h(q_2) + \Delta P\}$ where $\Delta P = P_1 - P_2$.

Thus, what determines the players choice, from the planner's perspective, is only ΔP and not the absolute value of P_1 and P_2. Since ΔP is a constant, the equilibrium condition in terms of cumulative distribution is

$$1 - F(h(q_1) - h(q_2) + \Delta P) = q_1$$

Using the same arguments presented in the proof of proposition 2, for any ΔP, a unique equilibrium exists. Notice that the *equilibrium* distribution depends on the prices. We are interested in looking for the function that links ΔP with the equilibrium distribution.

By now, we only know that we can associate to each ΔP an equilibrium distribution, i.e. we know that there is a function that links ΔP to an equilibrium distribution, but we can not yet assert, for instance, that there is a one-to-one relation between them. However, suppose that (q_1^*, q_2^*) is a fixed point and that both $\Delta P'$ and $\Delta P''$[1] imply (q_1^*, q_2^*).

Therefore, because of the monotonicity of $\Delta V(t)$, we have two distinct set $T1$ of the form $[t, 1]$, say $T1' = \{t \in T, \quad \Delta V(t) \geqslant h(q_1^*) - h(q_2^*) + \Delta P'\}$ and $T1'' = \{t \in T, \quad \Delta V(t) \geqslant h(q_1^*) - h(q_2^*) + \Delta P''\}$. But, since (q_1^*, q_2^*) is a fixed point, it has to be that $\lambda(T1') = q_1^* = \lambda(T1'')$. Thus $T1' = T1''$.

[1] Here, to avoid uninteresting results (i.e. full concetration of players on only one period) assume that ΔP is low enough. This assumption will be formalized in the sequel.

It clearly follows that $\Delta P' = \Delta P''$ and so for any equilibrium distribution there exists a unique ΔP in relation with it. Then, the relation between ΔP and $(q_1^*(\Delta P), q_2^*(\Delta P))$ is a bijection. Also, the strategy profile of equilibrium is univoquely defined by

$$f^*(t; \Delta P) = \begin{cases} 2 & \text{for } t < t^*_{\Delta P} \\ 1 & \text{for } t \geqslant t^*_{\Delta P} \end{cases}$$

where $t^*_{\Delta P} = q_2^*(\Delta P)$, given the form of $T1$ and $T \backslash T1$. We resume this argument in the following:

Proposition 3 *In the two stages game presented in this section, there is a one-to-one relation between the ΔP chosen by the first mover and the equilibrium distribution resulting by the strategic interaction of the continuum of second movers, i.e. $(q_1^*(\Delta P), q_2^*(\Delta P))$.*

Note two issues. First, from the social planner's perspective, there is a degree of freedom in choosing the prices because what matters is their difference and not their absolute value. Secondly, we do not care about the absolute utility level that each agent gets in equilibrium: it may also be negative, due to the prices.

To deal with the two above remarks, we made the following assumption:

Assumption 2 The social planner sets the prices that induce the first best distribution in such a way to minimize the size of his own intermediation.

Under this assumption, the prices pair is uniquely defined since it solves the program of minimizing $P_1 + P_2$, under the constraints that P_i, $i = 1, 2$, is nonnegative and that $P_1 - P_2 = \Delta P^{FB}$, where ΔP^{FB} is the particular value of ΔP inducing the equilibrium distribution of first best. The solution of this program is clearly $P_i = |\Delta P^{FB}|$ and $P_j = 0$, with $i \neq j$. More precisely, i equals 1 and j equals 2 if the first best allocation is such that $q_1^* > q_1^{FB}$ (and vice versa) where q_1^* is the "number" of those choosing the strategy 1 at the Nash equilibrium of the one-stage game without social planner and q_1^{FB} is the first component of the first best allocation chosen by the social planner. Notice also that having at least one zero price ensures that, at the equilibrium, any player gets a nonnegative utility.

Now, we are ready to solve completely the two-stage game and so to determine the first best allocation. Following the chain of the one-to-one relations presented, we have finally determined a one-to-one relation between any prices pair and any strategy profile of equilibrium.

Actually, there is a bijection between ΔP and the equilibrium distribution and a bijection between ΔP and (P_1, P_2), given assumption 2. The social planner's problem of determining the efficient prices is equivalent to choose the equilibrium strategy profile or simply the pivotal individual completely characterizing it.

Thus, the social planner chooses the $t^*_{\Delta P}$ that maximizes

$$\int_0^{t^*_{\Delta P}} u_2(t, t^*_{\Delta P})dt + \int_{t^*_{\Delta P}}^1 u_1(t, 1 - t^*_{\Delta P})dt$$

The prices payed by the agents are not wasted and simultaneously represent a negative and a positive component in the planner's objective function that completely off-set each other. A sufficient but not necessary condition for the strictly concavity of the above function is that $h(\cdot)$ is convex. The first order condition yields:

$$u_1(t^{FB}, q_1^{FB}) - u_2(t^{FB}, q_2^{FB}) = q_1^{FB} h'(q_1^{FB}) - q_2^{FB} h'(q_2^{FB}) \tag{2}$$

Since the equilibrium strategy profile is a Nash equilibrium, the condition of indifference for the pivotal individual of first best must be verified. His net utility is $u_i(t^{FB}, q_i^{FB}) - P_i$ for $i = 1, 2$ and the indifference condition results to be $u_1(t^{FB}, q_1^{FB}) - u_2(t^{FB}, q_2^{FB}) = \Delta P^{FB}$. Therefore, we have that

$$\Delta P^{FB} = q_1^{FB} h'(q_1^{FB}) - q_2^{FB} h'(q_2^{FB})$$

Let us analyze the economic meaning of the previous conditions. We have two symmetric cases included in the following:

Proposition 4 *Consider the first best distribution as determined by (2):*

- *if $t^{nash} < t^{FB}$, then there will be $t^{FB} - t^{nash}$ players that will be induced by the prices to change their strategy from 1 to 2. The optimal prices are $P_1 = \Delta P^{FB}$ and $P_2 = 0$. The pivotal individual of first best t^{FB} is such that his utility loss, due to the change of strategy, equals the gain enjoyed by those players remained in the period 1, due to the decreased congestion in period 1, minus the loss suffered by all the players in the period 2, due to the increased congestion in period 2.*

- *if* $t^{nash} > t^{FB}$*, then there will be* $t^{nash} - t^{FB}$ *players that will be induced by the prices to change their strategy from 2 to 1. The optimal prices are* $P_1 = 0$ *and* $P_2 = -\Delta P^{FB}$*. The pivotal individual of first best* t^{FB} *is such that his utility loss, due to the change of strategy, equals the loss suffered by all the players in the period 1, due to the increased congestion in period 1, plus the gain enjoyed by those players remained in the period 2, due to the decreased congestion in period 2.*

Shortly, the pivotal individual of first best is such that his loss of utility, after the planner's intervention, equals the aggregate externality on all the other players due to his change of strategy (i.e. of consumption period). Finally, it is worth to remark that the pivotal individual gets a null net utility at equilibrium, while all the others obtain a strictly positive payoff.

Positive prices are imposed to the peak period consumers. This is because the planner wants to move players from the more congested period towards the less congested, in order to get better as many agents as possible and, symmetrically, to get worse as less as possible. This implies that a quantity peak reversal situation is never optimal (as in Shy, 2001), i.e. the first best distribution presents the same peak period as the Nash equilibrium distribution, even if the former is more equilibrated than the latter. Hence, the social planner is only interested in an efficient allocation of the congestion losses over the whole set of players.

4 Two problems for a monopolist

Consider, now, the following two stage game: in the first stage a monopolist chooses the prices in order to maximize his own profit; in the second stage, a continuum of players observes the prices and chooses to consume in period 1, in period 2 or never. We deal with two situations: in the first, the monopolist is obliged to guarantee a nonnegative utility to each player, so that any player participates; in the second, the monopolist may exclude someone from the consumption.

4.1 Full participation

The second stage game, when the monopolist is constrained to guarantee a nonnegative utility, is like the game presented in the social planner's problem, because the strategy 'no participation' is dominated. Indeed, we can simply consider the same game, with two possible strategies, and the results stated in section 3. What remains is to explicit the participation constraint.

There are three relevant players' subsets: we call $T1$ the set of those players choosing period 1, given the monopoly prices; we call $T2$ the set of those choosing period 2, given the monopoly prices; we call T_j^i the set of players moving from period i to period j, because of the monopoly prices. Remember that, given the payoff specification, such three sets are convex, disjoints and form a partition for T.

We recall that the measure of the players' set choosing period i without prices is, in equilibrium, q_i^* (i.e. the $i-th$ component of the Nash equilibrium distribution, as determined in section 2) and the measure of the players' set choosing period i with the monopoly prices is, in equilibrium, $q_i^m < q_i^*$, because of the definition of T_j^i.

To simplify some notational problems, we separate two cases: before we analyze $q_2^m > q_2^*$ and after $q_2^m < q_2^*$.

Firstly, let be $q_2^m > q_2^*$.

To obtain the monopoly distribution (q_1^m, q_2^m) the monopolist has to set P_1 and P_2 such that, for all $t \in T_2^1 = [q_2^*, q_2^m]$,

$$V_1(t) - h(q_1^m) - P_1 \leqslant V_2(t) - h(q_2^m) - P_2$$

In fact, he wants that some players move from period 1 to period 2. Therefore the implementability condition amounts to be

$$\Delta P \geqslant \Delta V(t) - h(q_1^m) + h(q_2^m) \tag{3}$$

The sufficient condition so that (3) is valid for all $t \in T_2^1$ is that

$$\Delta P \geqslant \Delta V(t^m) - h(q_1^m) + h(q_2^m) \tag{4}$$

where $t^m = q_2^m$, i.e. it is sufficient that (3) is true at the $\sup(T_2^1)$, given $\Delta V(t)$ increasing.

The participation constraints are now discussed. We need a new assumption:

Assumption 3 $V_1(t)$ is increasing and $V_2(t)$ is decreasing, i.e. in the population, the covariance $cov(V_1, V_2)$ is negative.

We then suppose that among our population there are individuals with opposite valuation for day and night connection: someone assigns high importance to day (resp. night) and low value to night (resp. day) consumption. For instance, workers can browse (for their private pleasure or utility) only after their work-time, while firms pay extreme attention exactly to work-time connection.

However, it may also be reasonable (depending of the contexts that one studies) assuming a positive correlation. This would mean that connection (or consumption) has always high value for someone, or it has a value *per se,* independently of its timing (and vice versa low value for others). A student may appreciate browsing for research in the daytime as well sending e-mails and chatting at home in the evening. On the other hand, a heremit may attribute no value at all to internet.

We will assume this view in section 5.

For $t \in T_2^1$, if $t^m = q_2^m$ participates, then any other $t \in T_2^1$ will do, and he chooses period 2, because $V_2(t)$ is decreasing. Therefore, the participation condition is simply

$$P_2 \leqslant V_2(t^m) - h(q_2^m) \tag{5}$$

As explained in section 2 and 3, t^m is the pivotal individual in the monopolist game and he is indifferent between period 1 and period 2.

For $t \in T_1 =]q_2^m, 1]$, if $t^m = q_2^m$ has a nonnegative utility choosing period 1, then all others $t \in T_1$ will have a positive payoff and they will choose period 1, because $V_1(t)$ is increasing. Therefore, the participation condition is

$$P_1 \leqslant V_1(t^m) - h(q_1^m) \tag{6}$$

Finally, for $t \in T_2 = [0, q_2^*[$ the participation condition is

$$P_2 \leqslant V_2(t^*) - h(q_2^m)$$

that is verified as long as (5) is.

From (3), (5) and (6) we have that the monopoly distribution can be obtained only for

$$\Delta P = \Delta V(t^m) - h(q_1^m) + h(q_2^m)$$

Therefore, the highest possible prices that can be set by the monopolist, given (5) and (6) are

$$P_1 = V_1(t^m) - h(q_1^m) \quad \text{and} \quad P_2 = V_2(t^m) - h(q_2^m)$$

In the second case, i.e. $q_2^m < q_2^*$, some players have to move from period 2 to period 1.

For $t \in T_1^2 = [q_2^m, q_2^*]$, to obtain the monopoly distribution, the monopolist has to set P_1 and P_2 in such a way that

$$V_1(t) - h(q_1^m) - P_1 \geqslant V_2(t) - h(q_2^m) - P_2$$

Again, this condition is verified if it is true for $t^m = q_2^m$. The implementability condition becomes

$$\Delta P \leqslant \Delta V(t^m) - h(q_1^m) + h(q_2^m) \tag{7}$$

The participation conditions are:

1. for $t \in T_1^2$ the condition is
$$P_1 \leqslant V_1(t^m) - h(q_1^m) \tag{8}$$
2. for $t \in T_2$ the condition is
$$P_2 \leqslant V_2(t^m) - h(q_2^m) \tag{9}$$
3. for $t \in T_1$ the participation constraint is slack if (8) is verified.

Notice that (8) and (9) imply (7). Furthermore, the highest possible prices are defined as before

$$P_1 = V_1(t^m) - h(q_1^m) \quad \text{and} \quad P_2 = V_2(t^m) - h(q_2^m) \tag{10}$$

The optimal monopoly prices are such that the pivotal individual, whatever choice he makes, gets a null utility, i.e. the monopolist extracts the whole surplus from the pivotal individual, while he has to give up a strictly positive surplus to all others players. Differently from the social planner prices, here the concern is put on the surplus extractable and not on the congestion effect.

We are now ready to state and solve the problem of a monopolist that sets the prices in order to maximize his profit under the constraint that the whole set of players has to participate. This constraint implies that no demand restriction is available and so the best to do is to set the prices as high as possible.

As in the problem of the social planner, there is a bijection that links the prices and the equilibrium distributions: fixed the pair (P_1, P_2), as defined in (10), we get the unique equilibrium distribution (q_1^m, q_2^m). This result is directly implied by proposition 3 and by the fact that the monopolist maximizes his own profit. His profit function is

$$\pi(P_1, P_2) = P_1 q_1(\Delta P) + P_2 q_2(\Delta P)$$

Since for any ΔP the total demand is $q_1(\Delta P) + q_2(\Delta P) = 1$, the profit is maximal for (P_1, P_2) as high as possible, i.e. as defined in (10). Now, the only remained degree of freedom for the monopolist is to set the ΔP that yields the profit maximizing equilibrium distribution. Given the one-to-one function between ΔP and (q_1^m, q_2^m), the choice of ΔP is equivalent to the choice of the equilibrium strategy profile that maximizes the profits. We have already seen that any equilibrium strategy profile with prices has the form

$$f^m(t; \Delta P) = \begin{cases} 2 & \text{for } t < t_{\Delta P}^m \\ 1 & \text{for } t \geqslant t_{\Delta P}^m \end{cases}$$

where $t_{\Delta P}^m$ is the pivotal individual indifferent between the two periods.

Indeed, the monopolist's problem reduces simply to determine $t_{\Delta P}^m$ or

$$t_{\Delta P}^m = \arg\max \quad \{\pi(t^m) = [V_1(t^m) - h(q_1^m)](1 - t^m) + [V_2(t^m) - h(q_2^m)]t^m\}$$

The first order condition yields:

$$u_1(t_{\Delta P}^m, q_1^m) - u_2(t_{\Delta P}^m, q_2^m) = q_1^m u_1'(t_{\Delta P}^m) + q_2^m u_2'(t_{\Delta P}^m) \tag{11}$$

The economic interpretation of this condition is immediate. We include it in the following:

Proposition 5 *The pivotal individual $t_{\Delta P}^m$ is such that the utility variation he suffers for the passage from period 1 to period 2 equals the variation of the aggregate surplus the monopolist is able to extract from all other players, variation due to the $t_{\Delta P}^m$'s change of strategy. The optimal prices are defined in (10).*

The variation of the aggregate surplus is due to two distinct factors. To be simple, suppose that the monopoly prices induce some players to move from day to night. Players remained in period 1 are those with the highest $V_1(t)$, given the form of the sets T_1, T_2 and T_2^1, and so they can pay a higher price. This is the first factor that we call willingness to pay effect (WE), i.e. the pivotal player, as determined in monopoly, has a higher willingness to pay.

The second factor is due to the congestion (congestion effect, CE): those remained in period 1 suffer less congestion and so they enjoy a higher utility. Vice versa, the players' number in period 2 grows. These individuals have both a lower marginal $V_2(t)$ and suffers from a higher congestion. Therefore, the monopolist is able to increase P_1 but he has to decrease P_2 because the whole demand has to be satisfied.

In other words, to raise the day price it is necessary to expand night participation and the sole tool to do so is to decrease the night price. Because of the full participation constraint, it is not possible to isolate the players with the highest valuation either for period 1 or for period 2 and set both P_1 and P_2 to a higher level, as we will see possible in the following subsection. Indeed, the monopolist cannot operate a bilateral discrimination.

Let us rewrite (11) to explicit the two effects:

$$\Delta u(t^m_{\Delta P}, q_1^m, q_2^m) = [q_1^m V_1'(t^m_{\Delta P}) + q_2^m V_2'(t^m_{\Delta P})] + [q_1^m h'(1 - t^m_{\Delta P}) - q_2^m h'(t^m_{\Delta P})] \tag{12}$$

Both effects can be positive or negative and may offset or strengthen each other. Depending of the combination of WE and CE, the monopolist may either equilibrate the players' distribution, to extract the increased aggregate surplus, or may even widen the demand of the peak period, if the players choosing the off-peak have a quite high willingness to pay.

Furthermore, the quantity peak reverse phenomenon may occur, i.e. in monopoly the Nash equilibrium off-peak period may become the peak. This is because what matters is how much surplus can be extracted and not only how efficiently the congestion costs can be distributed.

Remark 1 *In any equilibrium profile the utility variation of the pivotal individual t^* (with or without prices) is a function of t^* only. Moreover, $\Delta u(t^*, q_1^* = 1 - t^*, q_2^* = t^*) = \Delta u(t^*) = \Delta V(t^*) - h(1 - t^*) + h(t^*)$ is increasing in t^*. In particular, the Nash equilibrium condition (1) becomes*

$\Delta u(t^{nash}) = 0$*; the Social Planner condition is* $\Delta u(t^{FB}) = CE^{FB}$*; the monopolist condition is* $\Delta u(t^{m}_{\Delta P}) = WE^m + CE^m$

If $WE^m + CE^m > 0$, the monopoly effect increases the peak demand if $t^{nash} > \frac{1}{2}$ and equilibrate the distribution if $t^{nash} < \frac{1}{2}$. In this last case we may have quantity peak reverse. Here, we present some examples:

Functions	t^{nash}	t^{FB}	$t^{m}_{\Delta P}$	peak reverse
$V_1 = t^2 + t$ $V_2 = 1 - t$ $h = t^2$	0,449	0,472	0,523	yes
$V_1 = t^2 + t$ $V_2 = 1 - t$ $h = t^2 + 2t$	0,472	0,485	0,513	yes
$V_1 = t^2 + 2t$ $V_2 = 1 - t$ $h = t^2$	0,372	0,424	0,519	yes
$V_1 = t^2 + 2t$ $V_2 = 1 - t$ $h = t^2 + 2t$	0,424	0,458	0,511	yes
$V_1 = t^2$ $V_2 = 2 - 2t$ $h = t^2$	0,646	0,583	0,523	no
$V_1 = t^2$ $V_2 = 2 - 2t$ $h = t^2 + 2t$	0,583	0,544	0,513	no
$V_1 = t^2 + t + 1$ $V_2 = \frac{3}{2} - t$ $h = t^2$	0,345	0,416	0,477	no

In the third example the difference between the distributions are striking: there are many players with a relatively high valuation for day; the Nash equilibrium is dominated and determined by this feature. The monopoly equilibrium is reversed because the monopolist finds optimal to extract as much surplus as possible from the players with the high day valuation.

The aggregate congestion loss can simply be calculated as $q_1 h(q_1) + q_2 h(q_2)$: it is 0,299 in the spontaneous Nash equilibrium, 0,267 in the social planner's distribution and only 0,251 in the monopolist's distribution (in fact the most equilibrated among the three, as in all the presented examples). Note also how an increased congestion function reduces the dispersion of the three presented distributions: the reader may appreciate it from the first example to the second, from the third to the fourth and from the fifth to the sixth. The reason is that a high congestion disutility reduces the players' heterogeneity in terms of private valuation.

4.2 Reduced participation

A more general setting is worth to discuss. In the first stage the monopolist chooses his prices and in the second stage the continuum of players chooses whether and when to participate. Each player can guarantee to himself at least a null utility by not participating. More formally the second stage is a static game, with complete information, where the strategy space contains three strategies: consume in period 1 (strategy 1), consume in period 2 (strategy 2), do not consume at all (strategy 3). The payoff functions are defined as in section 2 except for $u_3(t) = 0$. The best reply function is

$$B_t(q_1, q_2, q_3) = \begin{cases} 1 & \text{if } u_1(t, q_1) - P_1 \geqslant u_2(t, q_2) - P_2 \text{ and } u_1(t, q_1) - P_1 \geqslant 0 \\ 2 & \text{if } u_1(t, q_1) - P_1 < u_2(t, q_2) - P_2 \text{ and } u_2(t, q_2) - P_2 \geqslant 0 \\ 3 & \text{if } u_i(t, q_i) - P_i < 0 \quad \text{for } i = 1, 2 \end{cases}$$

We define:
$T1 = \{t \in T, \quad u_1(t, q_1) - P_1 \geqslant u_2(t, q_2) - P_2 \text{ and } u_1(t, q_1) - P_1 \geqslant 0\}$
$T2 = \{t \in T, \quad u_1(t, q_1) - P_1 < u_2(t, q_2) - P_2 \text{ and } u_2(t, q_2) - P_2 \geqslant 0\}$.
$T3$ is simply $T \backslash (T1 \cup T2)$. The Nash equilibrium conditions, using the same argument of section 2, are

$$\lambda(T1) = q_1 \quad \text{and} \quad \lambda(T2) = q_2$$

We now state the following:

Proposition 6 *In this game any best reply pure strategy profile (and so any equilibrium pure strategy profile) has the form*

$$f^m = \begin{cases} 2 & \textit{for } t \leqslant t_2^m \\ 1 & \textit{for } t \geqslant t_1^m \\ 3 & \textit{for } t_2^m < t < t_1^m \end{cases}$$

Proof. The proposition is equivalent to say that $T1$ is a set such that: $t \in T1 \Rightarrow t' > t \in T1$; $T2$ is a set such that: $t \in T2 \Rightarrow t' < t \in T2$ and $T3$ is such that: $t \in T3 \Rightarrow t \notin T1$ *and* $t \notin T2$. Now $t \in T1 \Leftrightarrow u_1(t) - P_1 \geqslant u_2(t) - P_2$. Since $u_1(t, \cdot)$ is increasing in t and $u_2(t, \cdot)$ is decreasing in t, whenever $t' > t$ we have $u_1(t', \cdot) > u_1(t, \cdot) > u_2(t, \cdot) > u_2(t', \cdot)$. Therefore, $t' \in T1$ and in particular $t = 1 \in T1$ if $T1$ is non empty. Symmetrically, we prove for $T2$. Finally, $T3$ can be written as $]\sup T2, \inf T1[$ with $\sup T2 = t_2^m$ and $\inf T1 = t_1^m$. The pivotal individual t_i^m is such that $u_i(t_i^m, q_i^m) - P_i = 0$. ■

We have now to prove the existence and the uniqueness of the Nash equilibrium in pure strategies. Existence is guaranteed by the continuity of the payoff function and by the measurability of the sets $T1$, $T2$, $T3$ (Rath, 1992).

Lemma 1 *Given (P_1, P_2), f^m is the unique equilibrium in pure strategies.*

Proof. Suppose that two equilibria exist. They are completely described by the pairs (t_1^m, t_2^m) and $(t_1^{m\prime}, t_2^{m\prime})$. We need that $u_2(t_2^m, t_2^m) - P_2 = 0$ as well as $u_2(t_2^{m\prime}, 1 - t_2^{m\prime}) - P_2 = 0$. Therefore, we have that $V_2(t_2^m) - V_2(t_2^{m\prime}) = h(t_2^m) - h(t_2^{m\prime})$. If $t_2^m > t_2^{m\prime}$, the lhs is negative and the rhs is positive, because $V_2(t)$ is decreasing and $h(\cdot)$ is increasing. If $t_2^m < t_2^{m\prime}$, the lhs is positive and the rhs is negative. Hence, it has to be that $t_2^m = t_2^{m\prime}$. In the same way we prove that $t_1^m = t_1^{m\prime}$. ■

Now, we can define the pair (P_1, P_2) that induces the monopoly equilibrium with reduced demand.

Lemma 2 *The unique pair (P_1, P_2) inducing the equilibrium allocation $(q_1^m$, q_2^m, $q_3^m)$ is*

$$P_1 = u_1(t_1^m, q_1^m) \quad \textit{and} \quad P_2 = u_2(t_2^m, q_2^m)$$

Proof. Suppose $P_1 < u_1(t_1^m, q_1^m)$. Then $\exists t \in T3$ such that $u_1(t, \cdot) > 0 > u_2(t, \cdot)$. This is a contradiction because such t should belong to $T1$. Suppose now that $P_1 > u_1(t_1^m, q_1^m)$. Then, $u_1(t_1^m, q_1^m) - P_1 < 0$ and so t_1^m should belong to $T3$ since $u_2(t_2^m, q_2^m) - P_2 < u_1(t_1^m, q_1^m) - P_1 < 0$. This is a contradiction. Similarly, we prove for P_2. ■

Given these two last results, we again have a one-to-one function between prices and equilibrium strategy profile. The monopolist's problem is simply that of choosing t_1^m and t_2^m to maximize the profit, i.e.

$$(t_1^{m*}, t_2^{m*}) = \arg\max \quad \{\pi(t^m) = [V_1(t_1^m) - h(q_1^m)](1 - t_1^m) + [V_2(t_2^m) - h(q_2^m)]t_2^m\}$$

the first order conditions give:

$$u_1(t_1^{m*}, q_1^{m*}) = u_1'(t_1^{m*}, q_1^{m*})q_1^{m*} \tag{13}$$

$$u_2(t_2^{m*}, q_2^{m*}) = -u_2'(t_2^{m*}, q_2^{m*})q_2^{m*} \tag{14}$$

Proposition 7 *The pivotal individuals (t_1^{m*}, t_2^{m*}) are such that the monopolist's loss, reducing the demand for period i of the individual t_i^{m*}, equals the marginal surplus that the monopolist can extract from all those still consuming in period i.*

In such a setting, the monopolist can reduce the demand for both periods. Then a higher price for period i simply induces some players to pass from Ti towards $T3$. In other words, $T3$ is a sort of buffer that absorbs all reduction in period i without consequences in period j, as was the case in the previous subsection.

A high supply level reduces the *per capita* surplus the monopolist can extract, because of the congestion effect and the lower valuation of the marginal consumer; nevertheless, many people pay for connecting. On the other hand, a low supply level increases the *per capita* surplus that can be extracted, because of a lower congestion and a higher valuation of the marginal consumer; symmetrically, few people pay for connecting.

Hence, given the preceding first order conditions, we can conclude that *congestion worsen the access reduction of monopoly,* because it reduces the price elasticity of the aggregate demand. In other words, increasing a price has two opposite effects on the individuals: it reduces their surplus, but, lowering the congestion, it makes the service more valuable.

5 Supply Side

In this section we add to our model the supply issues. Until now we have assumed that enough capacity was installed and that its fixed costs were negligible. This allowed us to focus only on the demand distribution.

On the contrary, the problem is now to decide the network size, knowing that we can direct the demand, using appropriate prices.

We deal now with a slightly different setting. Differently from section 4, we endow our continuum of agents with two increasing valuations (for both day and night consumption). As mentioned, we aim to represent the fact that connection *per se* has a high value for some people whereas other people attribute to it a low value. However, we keep the hypothesis that some people strictly prefer daily connection and others night connection. More formally, we suppose:

Assumption 3bis $V_1(t)$, $V_2(t)$ are increasing, i.e. in the population, the covariance $cov(V_1, V_2)$ is positive.

We keep $\Delta V(t)$ increasing; moreover, there exists $t \in [0,1]$ such that $\Delta V(t) = 0$.

Consider again a two stage game. In the second stage a continuum of agents chooses how to distribute in the network, provided that the agents' payoff is positive. Players' utility is an additive function of the individual's valuation, the congestion effect and the imposed price. An individual can always get a null utility by non connecting. Indeed, three actions are possible: day connection (strategy 1), night connection (strategy 2), no connection (strategy 3).

In the first stage of the game, either a benevolent social planner or a monopolist, maximizes his objective function by choosing prices (people distribution) and the network size.

Lemma 3 *In the second stage, there exists a unique Nash equilibrium strategy profile characterized by two pivotal agents t_1 and t_2:*

$$f(t) = \begin{cases} 1 & \textit{for } t \geqslant t_1 \\ 2 & \textit{for } t_2 < t < t_1 \\ 3 & \textit{for } t \leqslant t_2 \end{cases}$$

determined in such a way that

$$V_1(t_1) - h(1 - t_1) - P_1 = V_2(t_1) - h(t_1 - t_2) - P_2 \tag{15}$$

and

$$V_2(t_2) - h(t_1 - t_2) - P_2 = 0 \tag{16}$$

Proof. We show that any best response strategy profile has to be characterized by two pivotal individuals. Given any distribution (q_1, q_2, q_3), if t chooses strategy 1, then any $t' > t$ will choose strategy 1, being $h(q_1)$ and P_1 fixed and $V_1(\cdot)$ being increasing. On the other hand, if t chooses strategy 3, i.e. for t both strategy 1 and 2 present negative payoff, then any $t' < t$ will choose strategy 3, being $V_1(\cdot)$ and $V_2(\cdot)$ increasing.

Let the lower t choosing strategy 1 be t_1 and the higher t choosing strategy 3 be t_2. It is clear that $t_1 \geqslant t_2$. Otherwise, there should exist $t_1 < t < t_2$ receiving a positive payoff by playing strategy 1, while t_2 is assumed to obtain a negative payoff. This is impossible.

Being $t_1 \geqslant t_2$, we get that people choosing strategy 2 are located between t_2 and t_1.

Using Rath, 1992, we have that a pure strategy Nash Equilibrium exists. Any equilibrium has the structure presented above, because a Nash equilibrium is a particular best response strategy profile.

Let (t_1, t_2) be a Nash Equilibrium. We said that any $t > t_1$ prefers the strategy 1. Any $t_2 < t < t_1$ prefers strategy 2. Therefore, t_1 has to be indifferent between the two. The indifference condition is simply the (15). Moreover, $t_2 < t < t_1$ preferring strategy 2 to strategy 1, connects until when his payoff from strategy 2 is positive. Since any $t < t_2$ does not connect, t_2 has to be indifferent between strategy 2 (night connection) and strategy 3 (no connection). The indifference condition is represented by (16).

Suppose that there exist two pure strategy Nash equilibria and denote them simply with (t_1, t_2) and (T_1, T_2). Suppose $T_2 > t_2$. Now evaluating (16) at T_2 and t_2 and subtracting the latter from the former, we get $V_2(T_2) - V_2(t_2) = h(T_1 - T_2) - h(t_1 - t_2)$. Here, the left hand side (lhs) is positive, since $V_2(\cdot)$ is increasing. For the right hand side (rhs) to be positive a necessary condition is that $T_1 > t_1$. Assume the rhs positive. Now, evaluating (15) at T_1 and t_1 subtracting again the latter from the former, we get $\Delta V(T_1) - \Delta V(t_1) = h(1 - T_1) - h(1 - t_1) - [h(T_1 - T_2) - h(t_1 - t_2)]$. Here the lhs is positive because $\Delta V(\cdot)$ is increasing. Nevertheless, the rhs is negative because $h(\cdot)$ is increasing. Therefore T_2 cannot be greater than t_2.

On the other hand, if $T_2 < t_2$ the same procedure can be applied. This condition implies also $T_1 < t_1$ and a contradiction is obtained.

Indeed, a unique Nash equilibrium exists in this game. ■

This setting, with a reservation utility (strategy 3) and contiguous sets of players on the network, directly imposes the (Nash equilibrium) inducing distribution prices, as determined by the indifference conditions of both pivotal individuals. However, any equilibrium distribution is attainable with a unique prices pair, given the monotonicity of $V_i(\cdot)$ and $h(\cdot)$. Hence, as in sections 3 and 4, prices and pivots are interchangeable.

5.1 Social planner

Now we analyse the problem of a benevolent social planner that has to choose the network size and the inducing equilibrium prices, in order to maximize the social welfare. Formally:

$$\max_{t_1,t_2} \int_{t_1}^{1} [V_1(t) - h(1-t_1)]dt + \\ + \int_{t_2}^{t_1} [V_2(t) - h(t_1 - t_2)]dt - c \max[1-t_1, t_1 - t_2]$$

He installs a capacity just sufficient to satisfy the peak demand. Unitary fixed cost are c. A capacity larger than the peak demand would be unused and its value lost. This is represented by the last term of the social welfare function.

Consider the case $t_1 - t_2 > 1 - t_1$, i.e., period 2 is the peak. This inequality is the domain of a simplified maximization, where $\max[1-t_1, t_1-t_2]$ is substituted by $t_1 - t_2$. We assume again $h(\cdot)$ convex to make the objective function concave. The first order conditions (FOCs) are

$$\Delta u(t_1) = h'(1-t_1)(1-t_1) - h'(t_1-t_2)(t_1-t_2) - c$$

and

$$u_2(t_2) = h'(t_1 - t_2)(t_1 - t_2) + c$$

Because of the convexity of $h(\cdot)$, the rhs of the former FOC is negative. Therefore, we get that $u_1(t_1) < u_2(t_1)$. This means that, at the social solution, there should be more individuals in period 1 than without the planner's intervention (at which $\Delta u(t_1^{nash}) = 0$, t_1^{nash} representing the "spontaneous" pivotal agent). In other words, the first best solution tends, once more, to equilibrate the distribution. The social planner equilibrates the demands up to the point where the utility loss of the pivotal (marginal) consumer (who must be displaced from period 2 to period 1), net of his (negative) impact on all other agents due of his displacement (i.e.,

$h'(1-t_1)(1-t_1) - h'(t_1-t_2)(t_1-t_2))$, equals the costs saved by reducing the installed capacity by "one" unit ($-c$).

The second FOC shows that the second pivotal agent is such that his utility, net of his impact on the others ($h'(t_1-t_2)(t_1-t_2)$), i.e., his impact on the social welfare, equals the cost of installing "one" more unit of capacity.

Hence, both pivotal individuals are determined in order to exactly offset marginal social benefits and costs. Furthermore, when prices are introduced to induce the first best distribution, notice that peak consumers (those in period 2) pay for capacity[2]. Such prices are:

$$P_1 = h'(1-t_1)(1-t_1)$$

$$P_2 = h'(t_1-t_2)(t_1-t_2) + c$$

In the case $t_1 - t_2 \leqslant 1 - t_1$, i.e., period 1 is the peak, we get symmetric conditions: there are less individuals in the first best than in spontaneous distribution; the social planner tends to equilibrate the demands; the pivots are such that marginal social benefits and costs are offset; peak consumer pay for capacity.

Notice that in both cases, price peak reverse is never possible (i.e., the peak is always greater than the off-peak price; see Bailey and White, 1974, and Shy, 2001). Neither, quantity peak reverse is optimal.

Furthermore, it may be good to leave part of the capacity unused when the off-peak is period 2: this is the case if the utility of one more consumer is lower than his (negative) impact on the others. When period 1 is off-peak, at the optimum, capacity is always partially unused, because it is not efficient to displace one more individual from period 2 to period 1, and adding one more agent in period 2, picked from those not connected. Otherwise the optimality conditions would be violated: intuitively, this is because the former individual would get $u_1 < u_2$ and the latter would have u_2 too low.

We resume this discussion in the following:

Proposition 8 *Peak consumers pay for capacity. The social planner tends to equilibrate the distribution. Prices are set to get a distribution which equates marginal social costs and benefits. It is, in general, inefficient fulfill*

[2] Remember that prices have to make both pivots indifferent between their two relevant actions, i.e., t_1 indifferent between periods 1 and 2, and t_2 indifferent between period 2 and "no connection at all".

capacity in the off-peak period: in particular, when period 2 is off-peak, this is because of the congestion effect..

Finally, observe that prices are sufficient to completely cover the equipment costs.

5.2 Monopolist

Now we discuss the monopoly framework.

The monopolist's objective function (his profit) is represented by

$$\max_{t_1,t_2} P_1(t_1,t_2)(1-t_1) + P_2(t_1,t_2)(t_1-t_2) - c\max[1-t_1, t_1-t_2]$$

where P_1 and P_2 are functions of the pivotal individuals, determined in (15) and (16).

Obviously, also a monopolist installs the capacity exactly necessary to satisfy the peak demand.

There are two cases, as before. The first case is $1-t_1 \geqslant t_1-t_2$, i.e. day is the peak period. Simple computations give:

$$P_1(t_1,t_2) = (1-t_1)\left[\frac{\partial P_1}{\partial t_1} + \frac{\partial P_1}{\partial t_2}\right] + (t_1-t_2)\left[\frac{\partial P_2}{\partial t_1} + \frac{\partial P_2}{\partial t_2}\right] + c$$

$$P_2(t_1,t_2) = (1-t_1)\frac{\partial P_1}{\partial t_2} + (t_1-t_2)\frac{\partial P_2}{\partial t_2}$$

The second case is $1-t_1 < t_1-t_2$, i.e. night is the peak period. Simple computations give:

$$P_1(t_1,t_2) = (1-t_1)\left[\frac{\partial P_1}{\partial t_1} + \frac{\partial P_1}{\partial t_2}\right] + (t_1-t_2)\left[\frac{\partial P_2}{\partial t_1} + \frac{\partial P_2}{\partial t_2}\right]$$

$$P_2(t_1,t_2) = (1-t_1)\frac{\partial P_1}{\partial t_2} + (t_1-t_2)\frac{\partial P_2}{\partial t_2} + c$$

It is apparent that, at the Nash equilibrium distribution induced by the monopolist, the capacity costs are paid by the peak demand, as in the standard peak load pricing theory. What is remarkable, in the second case, is that P_1 may be higher that P_2, although P_2 is the price associated to the peak demand. This is more likely when c is low and when many individuals have $V_2 > V_1$. This feature is the price peak reverse phenomenon.

Here is a simple example:

Example 1 *Let* $V_1(x) = x$, $V_2(x) = \frac{1}{8}x + \frac{3}{4}$ *,* $h(x) = x$ *and* $c = \frac{1}{16}$*. Remark that just* $\frac{1}{7}$ *of people prefer day connection to night connection. The unique solution of the monopolist's problem is for* $t_1 = \frac{441}{572}$ *and* $t_2 = \frac{249}{572}$*. The Nash Equilibrium distribution is then* $(q_1 = \frac{131}{572}, q_2 = \frac{192}{572}, q_3 = \frac{249}{572})$*. Clearly the peak is period 2 (night). Prices inducing this distribution are* $P_1 = \frac{1}{2}$ *and* $P_2 = \frac{15}{32}$*(both higher than the marginal cost of installation c). Finally, the monopolist's profit is* $\frac{287}{1144} \sim 0,25$.

Two issues are worth to be discussed. Firstly, it is not possible to separately set a period price. For instance, if the monopolist wishes to increase night demand, he can not simply cut P_2. Accepting more night consumers (decreasing t_2) has an (negative) effect also on P_1, in order to obtain a new equilibrium distribution. But, a lower P_1 may induce some night consumer to switch towards day connection. The overall effect, especially on profits, is not easily computable.

Secondly, because of the congestion effect on payoffs, accepting only few consumers in a given period allows to extract from them more surplus as discussed in section 4. If $h(\cdot)$ depended also on the available capacity, this effect would be more relevant. However, also in this simplified setting it may be profitable keep part of the capacity unused in the off-peak period.

Proposition 9 *Peak consumers pay for capacity. Nevertheless, with low implant cost and a high peak valuation, the peak-reverse phenomenon may arise, i.e. off-peak is higher than peak price.*

6 Conclusions

The setting analysed in this paper makes the demand levels endogenous as well as the peak period. Congestion is the way to obtain this endogenity: the players' payoff depends negatively on the "number" of players consuming at the same time. This phenomenon is particularly important in services such as internet connection or road transport.

Firstly, we have analyzed the consumption of such goods, where the crowding level matters in the consumers' decisions. We have argued that prices, set by a social planner, are able to modify the equilibrium distribution and, hence, they allow to allocate the congestion costs better than the "spontaneous" and strategical choice of the players.

Thereafter, we have studied what kind of effects prices have in a monopoly context. In a first case, we have assumed that the monopolist maximizes his profit under the constraint of universal service. Several scenarios are possible depending on the chosen payoff functions. He may equilibrate the players' distribution or he may even increase the peak demand. Also the quantity peak reverse phenomenon may occur. However, what matters is always how much additional surplus he may extract by moving some players from a period to another.

In a second case, we have permitted supply restriction. We get that a well defined set of players can not access to the network, given the monopoly prices. Reducing access allows the monopolist to extract more from the remained individuals. Furthermore, the lower congestion increases their surplus and, therefore, it worsens the access reduction of monopoly.

We have also discussed a more general model, where the supply size is a constraint. We have supposed that either a social planner or a monopolist decides it. In both situations, peak consumers pay the installation costs, as in the traditional peak-load pricing literature. Under certain hypothesis, in monopoly we have price peak-reverse: off-peak is higher than peak price, although this last "embodies" the fixed costs.

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